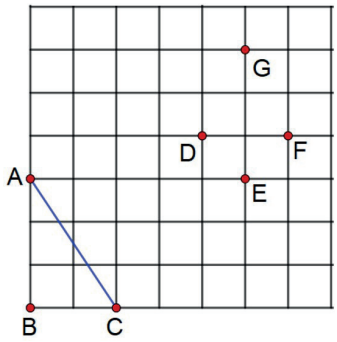
INTERNATIONAL STEM OLYMPIAD

GRAND FINAL

GRADES 9-10-11

MATHEMATICS PAST PAPERS

1.



The points A, B, C, D, E, F and G are displayed on a square grid.

Accordingly, which segment is perpendicular to segment [AC] ?

A) [BD]
B) [BE]
C) [BF]
D) [BG]

2.

What is the result of the following operation?

(1002·2001 - 1001·2002) ?

A) 999
B) 1000
C) 1001
D) 1999

3.

What is the sum of the roots of the following second degree equation?

$(x - 2)^2 + 4 \cdot (x - 2) - 32 = 0$

A) 0
B) 2
C) 4
D) 6

4.

F(n) is defined as : "the sum of the digits of n", with n being a natural number.

For example;

F(6)=6

F(51)=5+1=6

F(240)=2+4+0=6

F(1230)=1+2+3+0=6

Accordingly, what is the result of the operation F(10)+F(11) + ... + F(29)?

A) 120 B) 110 C) 105 D) 95

5.

The numbers a, b, and c are directly proportional to 6, 5, and 30, respectively.

Accordingly, which of the following options represents the numbers that are inversely proportional to a, b, and c, respectively?

A) 6, 5, 1

B) 5, 6, 1

D) 5, 1, 6

C) 6, 1.5

6.

Let A(x) and M(x) be defined as

A(x) = The sum of the numerals of x

M(x) = The product of the numerals of x

where "x" is a positive integer less than 1000.

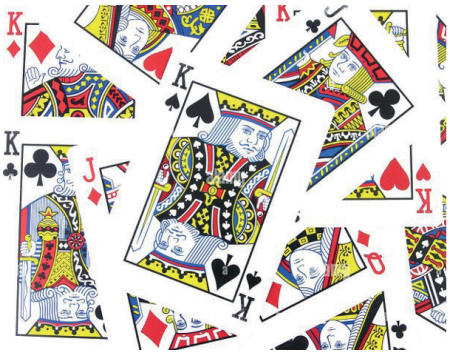
For example

A(234) = 2 + 3 + 4 = 9 and
M(x) = 2 · 3 · 4 = 24

What is the value of A(x) for the greatest three-digit integer that satisfies M(x) = 12 ?

A) 8 B) 9 C) 10 D) 11

7.



There are 52 cards in a deck, 12 of which are face cards (jack, queen, or king).

A deck of cards is shuffled. If 2 cards are drawn at random, what is the probability that both cards are face cards?

A) $\frac{11}{221}$ B) $\frac{1}{17}$ C) $\frac{9}{169}$ D) $\frac{1}{3}$

8.

480 students will be distributed into three different groups.

The number of students in the groups is directly proportional to the side lengths of a triangle.

Accordingly, which of the following triangles could be the mentioned triangle?

ΔABC: a = 3 cm, b = 5 cm, c = 7 cm

ΔDEF: d = 5 cm, e = 12 cm, m(F) = 90°

ΔJKL: j = 8 cm, k = 9 cm, l = 10 cm

A) ΔABC only

B) ΔDEF only

C) ΔABC and ΔDEF

D) ΔABC, ΔDEF and ΔJKL

9.

Three friends are going to order food based on the price brochure above.

Since the brochure is torn, they check the previous receipts and find that:

• 1 burger, 1 pizza, and 2 cokes cost 28 Euros.

• 1 hot dog, 1 pizza, and 1 coke cost 20 Euros.

Given so, how much will they pay in total if they order 3 hot dogs, 3 pommes, and 3 colas?

A) 39 Euros
B) 42 Euros
C) 45 Euros
D) 48 Euros

10.

If you increase the price of a product by 20% and then decrease it by 20% before selling it, what is the ultimate situation?

A) stability
B) 4% interest
C) 4% loss
D) 2% interest

11.

In a class of 25 students, 14 of them are girls. 4 of the girls and 6 of the boys are blond. What is the probability that a student selected from this class is a blond girl or a non-blond boy?

A) 24%
B) 30%
C) 36%
D) 42%

12.

1, 2, 3, 3, 3, 4, 4, 5, 5, 5, 6

Which of the following statements are true about the given number series?

I. Its mode is 3.

II. The sum of its median and range is 9.

III. Its arithmetic mean is an integer.

A) Only I

B) Only II

C) I and II

D) I, II, and III

13.

Bree has drawn hearts on a page of her math notebook in a pattern as described above.

If Bree continues in this way, how many hearts will there be on A_{30}?

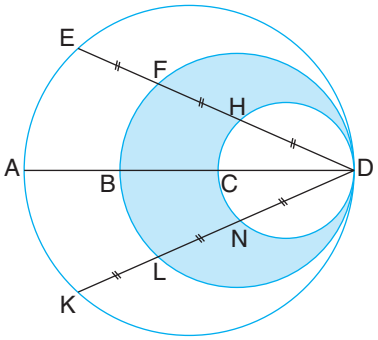
A) 676

B) 729

C) 900

D) 961

14.



William designed the target in the figure above to play darts. The target is formed of three circles that are tangent to each other at point D.

|KL|=|LN|=|ND|=|EF|=|FH|=|HD|

The line segments [AD], [BD] and [CD] are the diameters of the circles and |AD|=18 cm.

William makes a shot and it is known that he does not miss the target.

What's the probability that his shot hit the blue region?

A) $\frac{1}{9}$ B) $\frac{1}{3}$ C) $\frac{4}{9}$ D) $\frac{2}{3}$

15.

What is the value of x which satisfies the following equation

$$\frac{3}{x}+\frac{x-1}{4}=\frac{x}{4}-\frac{1}{x}$$

A) 2 B) 4 C) 8 D) 16

16.

What is the value of x which satisfies the following equation $5-\frac{2}{2+\frac{3}{x}}=3$

A) –3 B) –2 C) –1 D) 1

17.

- **Bingo is a game of chance in which each player matches numbers printed in different arrangements on cards with the numbers the game host draws at random, marking the selected numbers with tiles. When a player finds the selected numbers are arranged on their card in a row, they call out** "Bingo!" **to alert all participants to a winning card, which prompts the game host to examine the card for verification of the win.**

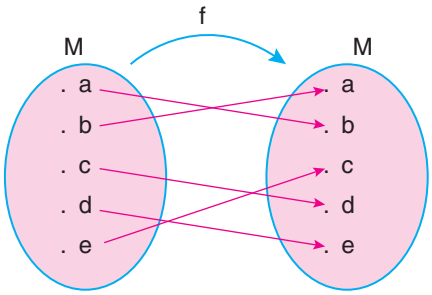
While playing Bingo Mrs Harrison notices that the numbers on the last line of her bingo card are five consecutive Fibonacci numbers. You can see the first and the last numbers in the picture below. The other three numbers are hidden.

B	I	N	G	O
3	28	35	55	65
14	27	36	52	68
6	20	FREE	58	75
4	19	71	54	61
5	■	■	■	34

If the sum of the first and fourth number on the last row is A, what's the sum of the second and third number on the same row in terms of A?

A) A–5 B) A+5 C) A–10 D) A+10

18.



The function f defined from M to M is given as above.

Given the circumstances, what is the value of (fofof)(c) ?

A) a B) b C) c D) d

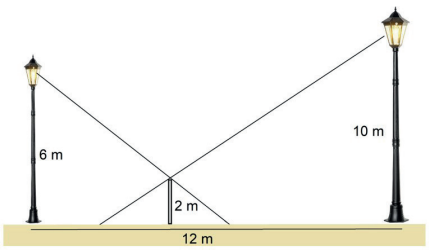
19.

Given that i = $\sqrt{-1}$ for the set of complex numbers, what is the value of the following?

$i^{24} + i^{25} + i^{26} + i^{27}$

A) i B) –i C) 0 D) 1

20.



Two lamp posts, 6 and 10 meters high, are positioned 12 meters apart on a straight path. A 2-meter bar is placed between the posts. The shadows formed by the lamps on both sides of the bar are equal.

Given that the posts and the bar are perpendicular to the ground, what is the length of the shadow created by one of the lamps?

A) 3,6 m B) 3 m

C) 2.4 m D) 2 m

21.

The functions f and g are defined as

f(m) = "number of positive divisors of the number g(m)"

g(x) = 4x + 2

Given so, what is the value of (fof)(2) ?

A) 4 B) 6 C) 8 D) 9

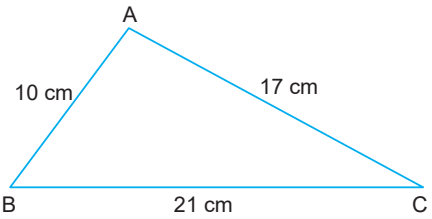
22.

A function "f" on the set of real numbers is defined as $f(mx+n) = \frac{m}{n}x + 1$.

Accordingly, what is the sum of f(0) + f(n)?

A) –1 B) 0 C) 1 D) 2

23.



In the figure above ABC is a triangle with |AB|= 10 cm |AC| = 17 cm and |BC| = 21 cm.

What is the value of $\frac{\tan(ABC)}{\tan(ACB)}$?

A) 3.5 B) 3 C) 2.5 D) 2

24.

Given that the numbers

$\log_a(8\cdot a)$, $\log_b(81\cdot b)$, and $\log_c(25/c)$

are all integers, with the numbers a, b, and c being positive integers,

which of the following cannot be the sum of (a+b+c) ?

A) 10 B) 20 C) 22 D) 36

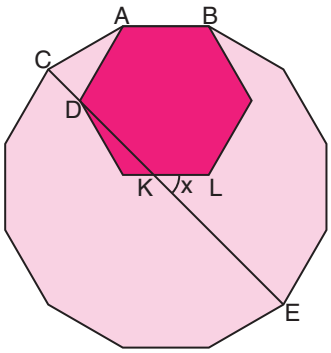
25.

A mother, a father, and their 5 children will sit side by side in a straight line.

What is the probability of two children sitting between the mother and the father?

A) $\frac{2}{21}$ B) $\frac{4}{21}$ C) $\frac{8}{21}$ D) $\frac{3}{7}$

26.



The measure of one of the interior angles of a regular polygon is evaluated by the formula $\frac{(n-2)\cdot 180}{n}$ where n is the number of sides of the polygon.

In the figure above the [AB] side of a regular hexagon and a regular dodecagon (a polygon with 12 sides) are coincident.

Given that the points C, D and E are linear, what is the measure of the angle LKE?

A) 45 B) 46 C) 48 D) 50

27.

We have the following information about the brick wall builders Diego and Pedro:

- **Diego can build a wall in 36 hours when he works alone. Pedro can build the same wall in 12 hours by himself.**
- **Pedro's wage per hour is twice Diego's wage per hour.**
- **When Diego and Pedro work together and finish building the wall, they get a wage of 270 $ in total.**

How much does Diego earn per hour?

A) 6$ B) 8$ C) 10$ D) 12$

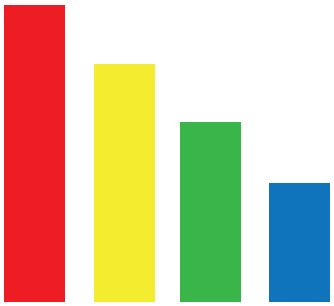
28.

The natural number n is the product of two prime numbers. The number of prime divisors of 66·n and 70·n are 5 and 3 respectively.

What's the sum of the digits of the natural number n?

A) 8 B) 9 C) 11 D) 14

29.



The heights of the rectangles given above increase by one centimeter from right to left. The height of the red rectangle on the left is $\log_3 24$ times the height of the blue rectangle on the right.

Accordingly, what is the height of the green rectangle?

A) $\log_2 3$ B) $\log_2 6$

C) $\log_3 10$ D) $\log_3 12$

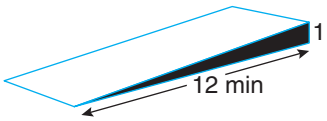
30.

What is the result of the following operation?

$$\sqrt{7 + 2\sqrt{12}} + \sqrt{12 - 2\sqrt{27}}$$

A) 3 B) $2\sqrt{3}$ C) 5 D) $3\sqrt{3}$

31.



A construction company makes all of their wheelchair-accessible ramps in a 12:1 ratio with regards to horizontal length:vertical height.

If a certain ramp needs to cover a vertical height of 1.5 meters, what will the length of the ramp be (to the nearest whole meter)?

A) 15 m B) 17 m C) 18 m D) 20 m

32.

In geometry, there are only 5 regular polyhedrons, also known as Platonic solids: tetrahedron, octahedron, icosahedron, cube, and dodecahedron. A regular polyhedron is defined as “a solid (convex) figure with all faces being congruent regular polygons, the same number arranged all alike around each vertex.”

An octahedron is made up of 8 congruent triangles meeting at 6 vertices. What is the sum of the angles at each vertex?

A) 220° B) 240° C) 260° D) 280°

33.

Let P = (1, -3, -4) and Q = (-5, 3, -1) on the three-dimensional coordinate system. What is a vector with the same direction as vector $\rightarrow$PQ and length of 3?

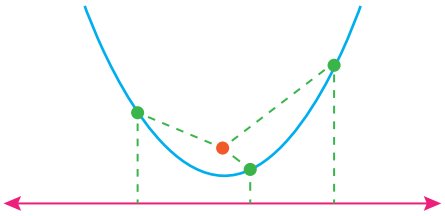
A) (-2, 2, 1)

B) (2, -2, -1)

C) (-6, 6, 3)

D) (6, -6, -3)

34.



A parabola can be defined as the set of all points which are equidistant from a certain point and a certain line. The point is called the focus and the line is called the directrix.

What is the equation of a parabola with its focus at (0, -5) and its directrix at x = -7?

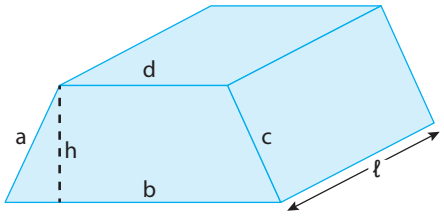
A) $x = \frac{1}{14} \cdot (y+5)^2 - 14$

B) $y = (x-5)^2 + 7$

C) $x = \frac{1}{14} \cdot (y+5)^2 - \frac{7}{2}$

D) $x = \frac{1}{7} \cdot (y-5)^2 - \frac{14}{5}$

35.



For the trapezoidal prism-shaped swimming pool above, given that a = c = 3 cm, b = 7 cm, d = 5 cm, and ℓ = 12 cm, what is the volume of water when the pool is filled to $\frac{2}{3}$ of its height?

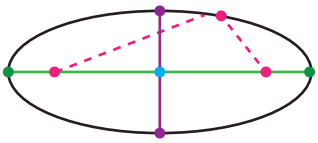
A) $(304\sqrt{2})/3$ cm^3

B) $(648-32\sqrt{2})/9$ cm^3

C) 72 cm^3

D) $(272\sqrt{2})/3$ cm^3

36.



An ellipse can be defined as the locus of all those points in a plane such that the sum of their distances from two fixed points in the plane is constant. These two points are called the foci (plural of focus).

What are the foci of the ellipse whose equation is

$26x^2 + 15y^2 + 312x - 90y + 681 = 0$?

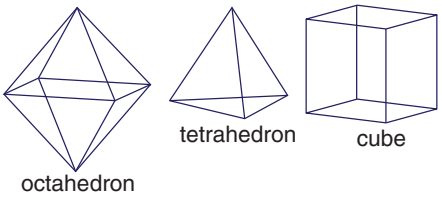
A) $(-6, 3 + \sqrt{451})$ and $(-6, 3 - \sqrt{451})$
B) $(-6, 3 + \sqrt{41})$ and $(-6, 3 - \sqrt{41})$
C) $(-6 + \sqrt{41}, 3)$ and $(-6 - \sqrt{41}, 3)$
D) $(-6 + \sqrt{451}, 3)$ and $(-6 - \sqrt{451}, 3)$

37.

What is the solution of the logarithmic equation, log(10x+20) - 2 log x = 1?

A) x = 2
B) x = -1
C) x = 1
D) x = 1 or 2

38.



A regular polyhedron is defined as “a solid (convex) figure with all faces being congruent regular polygons, the same number arranged all alike around each vertex.”

Which of the following geometrical structures has the highest surface area to volume (SA:V) ratio?

A) Cube
B) Regular Octahedron
C) Sphere
D) Regular Tetrahedron

39.

A farmer wants to build a rectangular enclosure for his horses. The enclosure has to be 175 square meters in area. The fencing for side AB costs $5.00 per meter while the fencing for the other three sides costs $2.00 per meter. What is the minimum cost for the farmer?

A) $137.50

B) $140.00

C) $165.00

D) $175.00

40.

A boy walks to school every day. If it is not raining, the probability that he is late is $\frac{1}{6}$. If it is raining, the probability that he is late is $\frac{1}{2}$. The probability that it rains on a particular day is $\frac{1}{4}$. On one particular day the boy is late. What is the probability that it was raining on that day?

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{7}{12}$