INTERNATIONAL STEM OLYMPIAD
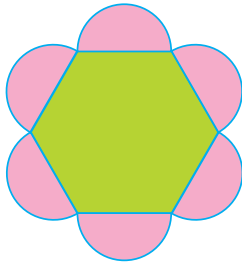
GRAND FINAL

MATHEMATICS PAST PAPERS

1.

The perimeter of the green regular hexagon is 24 cm and the sides of the hexagon are also the diameters of the pink semicircles.

Given the circumstances, what is the total area of the pink regions?

A) 8·π cm²   B) 10·π cm²

C) 12·π cm²   D) 18·π cm²

2.

A wheel with a diameter of 60 cm has traveled along a linear path from point A to point B, completing 25 full rotations.

Accordingly, what is the distance between points A and B?

A) 12·π m   B) 15·π m

C) 18·π m   D) 20·π m

3.

I- 7.24 · 25

II- 4.15 · 10

III- 100 · 4.6

IV- 8.4 · 10

Which of the multiplication operations given above have a result equal to a natural number?

A) I and III

B) II and III

C) I, III and IV

D) I, II and III

4.

When 6 boys and 12 girls leave a community of 46 people, the number of girls remaining is 6 times the number of boys remaining. Accordingly, how many girls were there in the community at the beginning?

A) 30   B) 32   C) 34   D) 36

5.

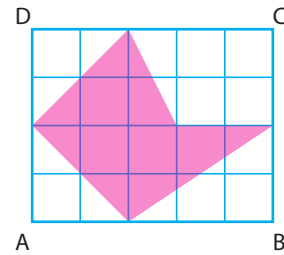What is the result of the operation
(112·231 - 111·232) ?

A) 120   B) 130   C) 140   D) 150

7.

In the figure above, ABCD is a rectangle divided into 20 equal squares with a side length of 1 cm.

What is the area of the shaded region?

A) 7   B) 8   C) 9   D) 10

6.

If you decrease the price of a product by 10%, and then later increase it by 10% before selling it, what is the ultimate situation?

A) stability

B) 2% interest

C) 2% loss

D) 1% loss

8.

What should be the smallest integer value of ▲ for the inequality below?

$$3\frac{4}{5} < \frac{▲}{15}$$
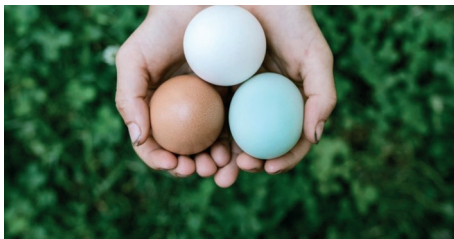
A) 55   B) 56   C) 57   D) 58

9.

How many distinct 2-digit numbers can be written using two non-zero numerals

A) 100

B) 90

C) 81

D) 72

11.

Allie, Cain, and Darcy are placing points on the number line starting from the zero point at equal intervals.

Allie places red points with intervals of 12 units, Cain places green points with intervals of 15 units, and Darcy places blue points with intervals of 20 units.

Based on this information, which point could have been marked with all three colors?

A) 160

B) 176

C) 180

D) 192

10.

There are 60 brown eggs, 72 white eggs, and 54 blue eggs that will be packed in egg-boxes in such a way that they will not mix, and each box will have an equal number of eggs in it.

How many boxes are needed at least for this process?

A) 24   B) 31   C) 34   D) 36

12.

While the thermometer in a car shows a temperature of 32°C, an air conditioner is turned on that can reduce the ambient temperature by 3°C per minute.

After the air conditioner is turned on for 4 minutes, what will be the temperature inside the car?
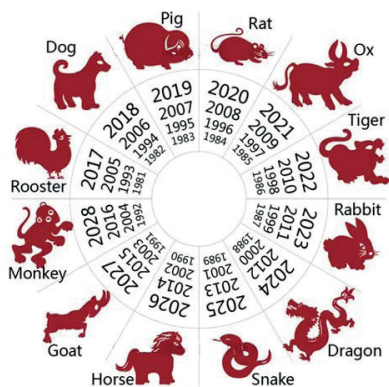
A) 16°C   B) 18°C   C) 20°C   D) 22°C

13.

What is the sum of the integers greater than −39 and less than 40?

A) 0   B) 1   C) 39   D) 40

14.

According to the Chinese Zodiac of 12 animals, what year will 2134 be?

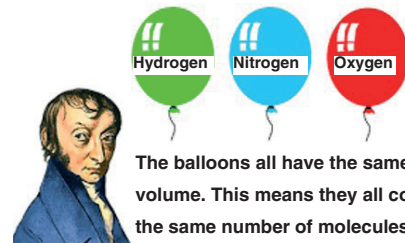A) Horse   B) Goat   C) Monkey   D) Rooster

15.

A coin is flipped 3 times. What is the probability of getting 3 heads?

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{3}{8}$ D) $\frac{1}{2}$

16.

The balloons all have the same volume. This means they all contain the same number of molecules.

Amedeo Avogadro

9 Aug 1776 – 9 Jul 1856

The number of units in one mole of any substance is called Avogadro's number. It is equal to $6.022140857 \cdot 10^{23}$

Based on the information above, how many zeros does Avogadro's number have at the end of it?

A) 13   B) 14   C) 15   D) 16

17.

There are 4 blue, 3 red and a number of yellow cars in a parking zone. The remote keys of all the cars are on a desk at the security office. The probability that a red car will beep is 25% when a security guard randomly grabs a remote car key and presses the unlock button.

How many yellow cars are there in the parking lot?

A) 2   B) 5   C) 8   D) 10

18.

Which of the following numbers is the greatest?

K = ($\sqrt{3}$ + $\sqrt{2}$) + ($\sqrt{3}$ – $\sqrt{2}$)

L = ($\sqrt{3}$ + $\sqrt{2}$) – ($\sqrt{3}$ – $\sqrt{2}$)

M = ($\sqrt{3}$ + $\sqrt{2}$) × ($\sqrt{3}$ – $\sqrt{2}$)

N = ($\sqrt{3}$ + $\sqrt{2}$) ÷ ($\sqrt{3}$ – $\sqrt{2}$)

A) K   B) L   C) M   D) N

19.

What is the result of the following operation?

$(\sqrt{54} - \sqrt{24}) \div \sqrt{2}$

A) $\sqrt{2}$ B) $\sqrt{3}$ C) $\sqrt{6}$ D) 2

20.

The knight is one of the six different types of chess pieces and is represented by a horse head. It is unique among chess pieces in that it moves in an L-shape pattern. Specifically, the knight moves two squares in a straight line horizontally or vertically, and then one square perpendicular to the first two, either to the left or right.

In a game of chess, the coordinates of a white knight is K(4, -3)

For which of the following black chessmen below is the white knight above not a threat?

A) (5,-1)

B) (3,-1)

C) (6,-3)

D) (3,-5)

21.

Given that A = 3x + 1 = 5y + 4 = 7z + 3, where x, y, and z are natural numbers, what is the smallest three-digit value of A?
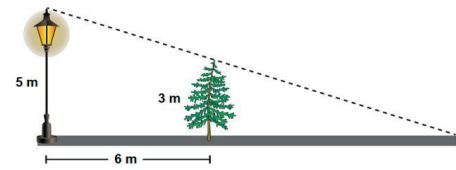
A) 176

B) 199

C) 234

D) 304

22.

1	2	3
4	5	6
7	8	9

Janet selects one number from each row randomly using the keys in the numeric keypad above to create her three-digit safe code.

According to this, what is the probability that all the numbers Janet selects are prime numbers?

A) $\frac{1}{27}$ B) $\frac{2}{27}$ C) $\frac{1}{9}$ D) $\frac{2}{9}$

23.

5 m   3 m   6 m

The distance between the 5-metre-tall lamp post and the 3-metre-tall tree depicted in the figure above is 6 metres.

Given that the lamp post and the tree are both perpendicular to the ground, what will be the length of the shadow cast by the tree when the lamp is lit?

A) 7 m   B) 8 m   C) 9 m   D) 10 m

24.

Door numbers of three secure vaults in a bank are 12, 27 and 28. Personnel numbers of four workers Adam, Betsy, Chloe and Drew are 45, 87, 55 and 54 respectively. A worker has access to a vault if the number on the door and his/her personnel number are relatively prime.

Which worker has access to all the vaults?

A) Adam   B) Betsy

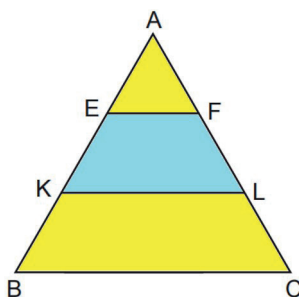C) Chloe   D) Drew

25.

There are 18 male and 12 female passengers on a bus. Two male and a female passengers get off the bus at a bus-stop, while 5 male and 3 female passengers get on it.

What is the probability that the first passenger to get off at the next bus-stop is male?

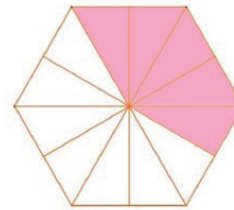A) $\dfrac{1}{3}$ B) $\dfrac{2}{5}$ C) $\dfrac{3}{4}$ D) $\dfrac{3}{5}$

26.

In the ABC triangle above
[EF] // [KL] // [BC], |AE| = |EK| = |KB| and
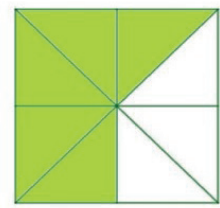|AF| = |FL| = |LC|.

What is the ratio of the trapezoid EKLF to the area of the triangle ABC.

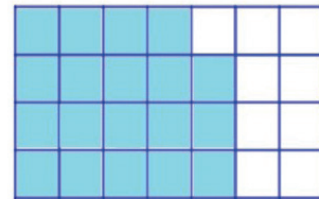A) $\dfrac{2}{3}$ B) $\dfrac{3}{5}$ C) $\dfrac{4}{9}$ D) $\dfrac{1}{3}$

27.

Sophie's puzzle **Dakota's puzzle**


Vanessa's puzzle

The puzzles of three students Sophie, Dakota and Vanessa have congruent masses.

Sophie's puzzle has 12 congruent pieces, while Dakota's have 8 and Vanessa's have 28 congruent pieces.

Which of the following options show the correct sorting for the undone pieces of girls' puzzles?

A) Sophie > Vanessa > Dakota

B) Vanessa > Sophie > Dakota

C) Sophie > Dakota > Vanessa

D) Dakota > Vanessa > Sophie

28.
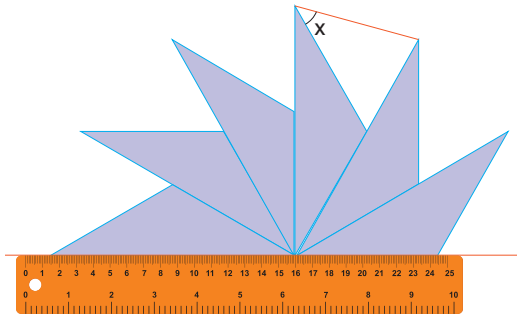
The purple isosceles triangles in the figure above are congruent. They are located tip to tip over a ruler as shown in the figure.

Given the circumstances, what is the measure of the angle x?

A) 30°   B) 35°   C) 40°   D) 45°

29.

The average of the ages of three friends was 20 when they first met at university.

Years later these three friends came together again, bringing their children to the meeting. The average of the ages of the six people at the meeting was 20.

Given that the age difference between the three friends and their children is 28, 30 and 32, after how many years did they meet?

A) 15   B) 16   C) 18   D) 20

30.
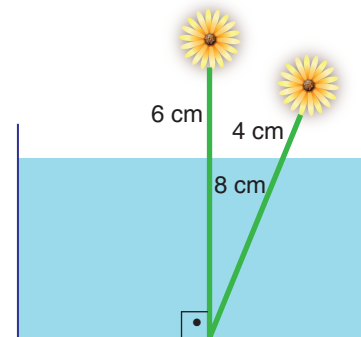
The roots of the flower in the figure above are under the water. Its linear stem which stands perpendicular to the ground changes its position because of the wind. In the second position the stem is still linear.

The part of the stem out of the water is 6 cm in its initial position and the part of the stem out of the water in its second position is 4 cm.

Given that the distance between the stems on the surface of the water is 8 cm, what is the length of the stem?

A) 19 cm   B) 20 cm
C) 21 cm   D) 22 cm

31.

Aborigines built tents with a cloth or animal skin thrown over a tent pole and then secured the cloth to the ground with stakes, forming an isosceles triangle. How long did the cloth have to be if they wanted the opening of the tent to be 4 meters high and 6 meters wide?

A) 5 m   B) 8 m

C) 10 m   D) $5\sqrt{2}$ m

32.

The Pentagon Building in the USA is essentially a pentagonal prism.

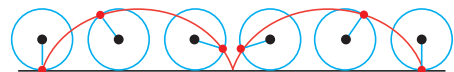**How many edges does a pentagonal prism have?**

A) 10   B) 12   C) 15   D) 20

33.

In the three-dimensional coordinate plane, what is the distance between the points (4, -3, -6) and (-2, 1, -4)?

A) $3\sqrt{13}$
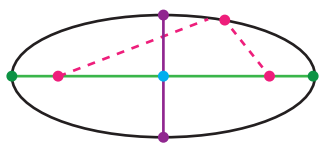
B) $2\sqrt{10}$

C) $4\sqrt{26}$

D) $2\sqrt{14}$

34.

A cycloid is the curve generated by a point on the circumference of a circle with radius r rolling along a straight line without slipping. Above are pictured two humps of a cycloid. What are the vertical height and horizontal length, respectively, of each hump?

A) 2r and 2πr

B) 2r and 3πr

C) 3r and 2πr

D) 3r and 3πr

35.

An ellipse can be defined as the locus of all those points in a plane such that the sum of their distances from two fixed points in the plane is constant.

What is the center of the ellipse whose equation is
$$12x^2 + 13y^2 - 144x + (26\sqrt{7})y + 367 = 0?$$

A) $(6, -\sqrt{7})$

B) $(-12, 13)$

C) $(-6, \sqrt{7})$

D) $(6, -2\sqrt{7})$

37.

What is the equation of a parabola with (0, 1) as its vertex and (0, 5) as its focus?

A) $y = x^2 + 1$

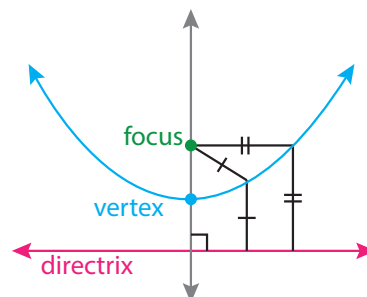B) $y = x^2/16 + 1$

C) $y = (x+1)^2/16$

D) $y = (x^2 + 1)/4$

36.

In geometry, there are only 5 regular polyhedrons, also known as Platonic solids: tetrahedron, octahedron, icosahedron, cube, and dodecahedron. A regular polyhedron is defined as "a solid (convex) figure with all faces being congruent regular polygons, the same number arranged all alike around each vertex."

An octahedron is made up of 8 congruent triangles meeting at 6 vertices. What is the sum of the angles at each vertex?

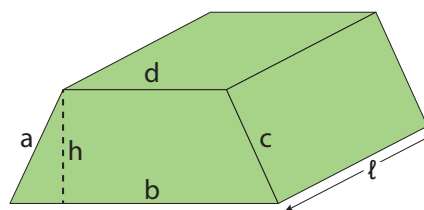A) 220°   B) 240°

C) 260°   D) 280°

38.

For the trapezoidal prism above, what is the volume given that a = c = $\sqrt{2}$ cm, b = 7 cm, d = 5 cm, and ℓ = 12 cm?

A) $54\sqrt{2}$ cm^3

B) $60\sqrt{2}$ cm^3

C) 72 cm^3

D) 144 cm^3

39.

What is the solution to the exponential equality $9^{3x} = 27^{(2-x)}$?

A) $x = \dfrac{1}{2}$

B) $x = \dfrac{2}{3}$

C) $x = \dfrac{3}{2}$

D) $x = 2$

40.

There are 3 apples and 7 oranges in a bag. One of the fruits is taken out of the bag at random and eaten. Then a second fruit is taken out, also at random, and eaten. What is the probability that one apple and one orange were eaten?

A) $\dfrac{2}{5}$ B) $\dfrac{1}{2}$ C) $\dfrac{7}{15}$ D) $\dfrac{4}{7}$